Random matrix theory: a review and new results

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Matrices with elements drawn randomly from statistical distributions are called "random matrices". It has been established that physical properties of many disordered systems (such as amorphous materials, a magnetic alloy like spin glasses, etc.) are determined by mathematical properties of random matrices, in particular, by their eigenvectors and eigenvalues [1,2,3]. One of the most famous results is the so-called Wigner’s semicircular law [1] which states that the average density $\rho(x)$ of the eigenvalue $x$ of a $N \times N$ symmetric real random matrix $(a_{ij})$ in the limit of matrix size $N \to \infty$ is

$$
\rho(x) = \begin{cases} 
\frac{\sqrt{4\sigma^2 - x^2}}{2\pi\sigma^2} & \text{if } |x| < 2\sigma \\
0 & \text{otherwise},
\end{cases}
$$

where each element $a_{ij}$ is drawn from independent identical distribution with zero odd-order moments, finite even-order moments and variance $\sigma^2$. This theorem was applied by May [5] to a linear stability analysis for a system with random interactions which exhibits a sharp transition from stable to unstable behavior when $N$ (diversity) or the typical interaction strength $\sigma$ (complexity) exceeds a critical value, the phenomenon of which is first discovered numerically by Gardner and Ashby [4]. I will first sketch out their results and the succeeding controversy on the stability of a large and complex ecosystem which is well known as the "paradox of ecology", and secondly give some recent examples of applications of random matrices to mathematical biology.

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